

# DVCS amplitude in the parton model

M. Penttinen<sup>a</sup>, M.V. Polyakov<sup>a,b</sup>, A.G. Shuvaev<sup>b</sup>, M. Strikman<sup>c</sup>

<sup>a</sup>*Institut für Theoretische Physik II, Ruhr-Universität Bochum,  
D-44780 Bochum, Germany*

<sup>b</sup>*Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia*

<sup>c</sup>*Department of Physics, Pennsylvania State University, University Park, PA 16802, USA*

## Abstract

We compute amplitude of deeply virtual Compton scattering in the parton model. We found that the amplitude up to the accuracy  $O(1/Q)$  depends on new skewed parton distributions (SPD's). These additional contributions make the DVCS amplitude explicitly transverse.

## Introduction

Hard exclusive processes, such as deeply virtual Compton scattering (DVCS), owing to the QCD factorization theorems [1, 2, 3, 4] allow to probe so-called skewed parton distributions [1, 3, 5, 6, 7].

In this note we shall study DVCS amplitude in the parton model, our prime interest will be the question what kind of skewed parton distributions enter DVCS amplitude up to the order  $O(1/Q)$ . Our calculations follow closely ideas of Ref. [8]. We compute the DVCS amplitude in the parton model, where the scattering of virtual photon occurs on a single parton on the mass-shell. In this approach the scattering amplitude is explicitly gauge invariant (transverse). We show that in the parton model in the case of pion target we reproduce results of Ref. [8]. Also we give an expression for the DVCS amplitude for an arbitrary target.

We shall see that even in the simplest parton model the handbag contribution to the DVCS amplitude depend not only on the matrix elements of the light-ray operators with indices projected on the light cone direction but also on the operators projected on the transverse direction. The corresponding additional contributions make the DVCS amplitude explicitly transverse. The additional terms in the DVCS amplitude are proportional either to the transverse component of the momentum transfer or the transverse component of the target polarization, as it follows from the recent operator analysis of Ref. [9].

Let us emphasize that calculations in the parton model we perform here correspond to the QCD calculation of the handbag diagram in time ordered perturbation theory in the infinite momentum frame. Therefore our calculations are model independent despite the use of the approximation of the parton model. The advantage of such formalism is that at each step of the calculations the amplitude is transverse (e.m. gauge invariant). Of course,

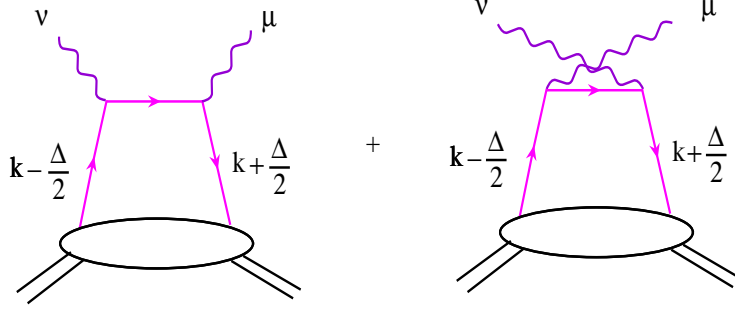


Figure 1: Handbag contribution to DVCS amplitude

the calculations in the parton model do not allow us to determine the contributions of the operators of the type  $\bar{\psi}G\psi$  to the amplitude. In order to determine contributions of such operators one has to perform the QCD operator analysis.

## DVCS amplitude

Here we give an expression for the handbag contribution to the DVCS amplitude computed in the parton model. The corresponding diagrams are shown in Fig. 1. It is convenient to introduce the light-cone decomposition of the momenta:

$$\begin{aligned}
 p^\mu &= (1 + \xi)\tilde{n}^\mu + (1 - \xi)\frac{\bar{M}^2}{2}n^\mu - \frac{1}{2}\Delta_\perp^\mu \\
 p'^\mu &= (1 - \xi)\tilde{n}^\mu + (1 + \xi)\frac{\bar{M}^2}{2}n^\mu + \frac{1}{2}\Delta_\perp^\mu \\
 q^\mu &= -2\xi\tilde{n}^\mu + \frac{Q^2}{4\xi}n^\mu \\
 \Delta^\mu &= (p' - p)^\mu \\
 \bar{M}^2 &= M_N^2 - \frac{\Delta^2}{4}.
 \end{aligned} \tag{1}$$

Here  $\tilde{n}^\mu$  and  $n^\mu$  are arbitrary light-cone vectors such that  $\tilde{n} \cdot n = 1$  and  $\tilde{n} \cdot \Delta_\perp = n \cdot \Delta_\perp = 0$ . The momentum  $k$  can be decomposed as follow:

$$k^\mu = x\tilde{n}^\mu + \beta n^\mu + k_\perp^\mu, \tag{2}$$

where  $\beta$  is fixed by the mass-shell conditions for the partons. Let us note that we do not need here the explicit expression for  $\beta$  because it contributes to the DVCS amplitude at the order  $O(\Delta_\perp^2/Q^2)$ . We are interested here in the order  $O(\Delta_\perp/Q)$ .

Computing scattering of the virtual photon on the mass-shell parton and neglecting terms of the order  $O(\Delta_\perp^2/Q^2)$  and  $M^2/Q^2$  one gets the following expression for the DVCS amplitude in the parton model:

$$M^{\mu\nu} \propto \int_{-1}^1 dx \int \frac{d^2 k_\perp}{(2\pi)^2} \int d^4 z \delta(n \cdot z) \frac{1}{(x - \xi)(x + \xi)} \exp[i(k \cdot z)] \left\{ \text{Tr} \left[ \left( \not{k} + \frac{1}{2} \not{\Delta} \right) \Gamma^{\mu\nu} \left( \not{k} - \frac{1}{2} \not{\Delta} \right) \not{n} \right] \langle p' | \bar{\psi}(-\frac{z}{2}) \not{n} \psi(\frac{z}{2}) | p \rangle \right. \\ \left. - \text{Tr} \left[ \left( \not{k} + \frac{1}{2} \not{\Delta} \right) \Gamma^{\mu\nu} \left( \not{k} - \frac{1}{2} \not{\Delta} \right) \not{n} \gamma_5 \right] \langle p' | \bar{\psi}(-\frac{z}{2}) \not{n} \gamma_5 \psi(\frac{z}{2}) | p \rangle \right\} + O\left(\frac{\Delta_\perp^2}{Q^2}\right), \quad (3)$$

where

$$\Gamma^{\mu\nu} = \frac{\gamma^\mu (\not{k} - \not{\Delta}/2 + \not{q}) \gamma^\nu}{(k - \Delta/2 + q)^2 + i0} + \frac{\gamma^\nu (\not{k} + \not{\Delta}/2 - \not{q}) \gamma^\mu}{(k + \Delta/2 - q)^2 + i0}. \quad (4)$$

Obviously, the amplitude (3) is transverse, *i.e.*  $(q - \Delta)^\mu M^{\mu\nu} = q^\nu M^{\mu\nu} = 0$ . In the limit  $Q^2 \rightarrow \infty$  and  $\Delta_\perp^2 \ll Q^2$  the expression (3) can be rewritten as:

$$M^{\mu\nu} = \frac{1}{2} \int_{-1}^1 dx \left\{ \alpha^+(x, \xi) \left[ (\tilde{n}^\mu n^\nu + \tilde{n}^\nu n^\mu - g^{\mu\nu}) - \frac{4\xi}{Q^2} \Delta_\perp^\nu \tilde{n}^\mu \right] n^\alpha F_\alpha \right. \\ + i\alpha^-(x, \xi) \left[ \varepsilon_\perp^{\mu\nu} + \frac{4\xi}{Q^2} \tilde{n}^\mu \varepsilon_\perp^{\Delta\nu} \right] n^\alpha F_\alpha^{(5)} \\ + \left[ n^\nu + \frac{8\xi^2}{Q^2} \tilde{n}^\nu \right] \left( \alpha^+(x, \xi) (F^{\mu\perp} + \frac{4\xi}{Q^2} \tilde{n}^\mu (\Delta_\perp \cdot F)) \right. \\ + i\alpha^-(x, \xi) (\varepsilon_\perp^{\mu\rho} F_\rho^{(5)} + \frac{4\xi}{Q^2} \tilde{n}^\mu \varepsilon_\perp^{\Delta\rho} F_\rho^{(5)}) \\ \left. \left. + n^\mu \left( \alpha^+(x, \xi) F^{\nu\perp} - i\alpha^-(x, \xi) \varepsilon_\perp^{\nu\rho} F_\rho^{(5)} \right) \right\} + O\left(\frac{\Delta_\perp^2}{Q^2}\right). \quad (5)$$

Here  $\varepsilon_\perp^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} n_\alpha \tilde{n}_\beta$ . We also defined:

$$\alpha^\pm(x, \xi) = \frac{1}{x - \xi + i0} \pm \frac{1}{x + \xi - i0}.$$

Skewed distributions  $F_\mu$  and  $F_\mu^{(5)}$  are defined in terms of bilocal quark operators on the light-cone<sup>1</sup>:

$$F_\mu = \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p' | \bar{\psi}(-\frac{\lambda}{2}n) \gamma_\mu \psi(\frac{\lambda}{2}n) | p \rangle, \\ F_\mu^{(5)} = \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p' | \bar{\psi}(-\frac{\lambda}{2}n) \gamma_\mu \gamma_5 \psi(\frac{\lambda}{2}n) | p \rangle. \quad (6)$$

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<sup>1</sup> The path ordered gauge link is assumed here. Although in the parton model we do not have interactions with the gluon fields the corresponding P-exponential can be obtained using eikonal approximation for the quark propagator in the external gluon field.

Generically, the expression (3) contains operators off the light-cone, but all of them can be reduced to the operators (6) using obvious operator identities like:

$$\bar{\psi}(y) \left[ \gamma_\alpha \nabla_\beta - \gamma_\beta \nabla_\alpha \right] \psi(x) = i \varepsilon_{\alpha\beta\rho\sigma} \bar{\psi}(y) \gamma_\rho \gamma_5 \nabla_\sigma \psi(x), \quad (7)$$

which are satisfied owing to the QCD equation of motion  $\nabla\!\!\!\!\!\nabla\psi(x)$ .

Note that two first terms in the expression (5) coincide exactly with the improved DVCS amplitude used in [10]. This part of the amplitude is characterized by leading-twist skewed parton distributions  $n \cdot F$  and  $n \cdot F^{(5)}$ , other contributions are characterized by new additional skewed parton distributions  $F_{\mu\perp}$  and  $F_{\mu\perp}^{(5)}$ . The latter functions are suppressed in differential cross section of the reaction  $e + N \rightarrow e' + \gamma + N$  by only one power of the hard scale  $1/Q$  [12, 13], therefore their estimates are important for the analysis of DVCS observables. In principle, considering azimuthal angle and  $Q$  dependencies of the various spin and charge asymmetries one should be able to disentangle the contributions of the additional SPD's from the leading twist ones [12]. Note also that in certain spin and azimuthal asymmetries the new functions enter at the same order in  $1/Q$  as the leading twist one, as examples of such quantities are  $\sin(2\phi)$  term in the lepton spin asymmetry, and *const* and  $\cos(2\phi)$  term in the lepton charge asymmetry. The measurements of such observables would give us an information on the size of new SPD's.

It is easy to check that the amplitude (5) explicitly satisfies the transversality conditions  $(q - \Delta)^\mu M^{\mu\nu} = q^\nu M^{\mu\nu} = 0$ . In the case of the pion target our expression (5) coincides with the result obtained by Anikin *et al.* [8]. For the nucleon target we can write for skewed parton distributions  $F^\mu$  and  $F^{\mu(5)}$ , for instance, the following decomposition:

$$F^\mu = \bar{u}(p') \left\{ \gamma^\mu H(x, \xi, \Delta^2) + \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M_N} E(x, \xi, \Delta^2) + \frac{\Delta_\perp^\mu}{M_N} \left[ G_1(x, \xi, \Delta^2) + M_N \not{n} G_2(x, \xi, \Delta^2) \right] + \gamma_\perp^\mu G_3(x, \xi, \Delta^2) + \dots \right\} u(p), \quad (8)$$

$$F^{\mu(5)} = \bar{u}(p') \left\{ \gamma^\mu \gamma_5 \tilde{H}(x, \xi, \Delta^2) + \frac{\Delta_\perp^\mu}{2M_N} \gamma_5 \tilde{E}(x, \xi, \Delta^2) + \frac{\Delta_\perp^\mu}{M_N} \gamma_5 \tilde{G}_1(x, \xi, \Delta^2) + \gamma_\perp^\mu \gamma_5 \tilde{G}_2(x, \xi, \Delta^2) + \Delta_\perp^\mu \not{n} \gamma_5 \tilde{G}_3(x, \xi, \Delta^2) + \dots \right\} u(p), \quad (9)$$

where ellipses stand for terms which do not contribute to the order  $1/Q$ . The two first functions in above equations coincide with the distributions introduced in Ref. [7]. The functions  $G_i(x, \xi, \Delta^2)$   $\tilde{G}_i(x, \xi, \Delta^2)$  are additional functions parameterizing the nucleon matrix element of quark light-cone operator. Obviously, these new functions satisfy the following sum rule:

$$\int_{-1}^1 dx \tilde{G}_i(x, \xi, \Delta^2) = 0, \quad \int_{-1}^1 dx G_i(x, \xi, \Delta^2) = 0. \quad (10)$$

The additional function  $G_1(x, \xi, \Delta^2)$  receive a contribution from the so-called D-term in the double distributions parametrization of light-ray operators [11]. This contribution to the function  $G_1(x, \xi, \Delta^2)$  has the form:

$$G_1(x, \xi, \Delta^2) = \frac{1}{2\xi} D\left(\frac{x}{\xi}, \Delta^2\right). \quad (11)$$

In the forward limit the SPD's  $F_\mu$  and  $F_\mu^{(5)}$  are:

$$\begin{aligned} F_\mu &\rightarrow 2\{f_1(x)\tilde{n}_\mu + M_N^2 f_4(x)n_\mu\} \\ F_\mu^{(5)} &\rightarrow 2\{g_1(x)\tilde{n}_\mu (n \cdot S) + g_T(x)S_{\perp\mu} + g_3(x)M_N^2 n_\mu (n \cdot S)\}. \end{aligned}$$

Hence, we observe that in the forward limit the SPD's  $F_\mu$  and  $F_\mu^{(5)}$  are reduced to the combinations of twist-2 parton distributions and higher twist distributions  $f_4(x)$ ,  $g_3(x)$  and  $g_T^{tw3}(x)$ . The functions  $F_\mu$  and  $F_\mu^{(5)}$  can be computed in the chiral quark-soliton model using methods of Refs. [14].

Very interesting sum rules can be derived if we consider the second Mellin moments of the distributions  $G_i$  and  $\tilde{G}_i$ . With help of identities like (7) one can derive relations between the second Mellin moments. Here we restrict ourselves to the following relations:

$$\int_{-1}^1 dx \, x \, G_3(x, \xi) = -\frac{1}{2} \int_{-1}^1 dx \, x \left[ H(x, \xi) + E(x, \xi) \right] + \frac{1}{2} \int_{-1}^1 dx \, \tilde{H}(x, \xi), \quad (12)$$

$$\int_{-1}^1 dx \, x \, \tilde{G}_2(x, \xi) = -\frac{1}{2} \int_{-1}^1 dx \, x \, \tilde{H}(x, \xi) + \frac{\xi^2}{2} \int_{-1}^1 dx \left[ H(x, \xi) + E(x, \xi) \right]. \quad (13)$$

The first relation in the forward limit can be related to the quark orbital momentum because:

$$\lim_{\Delta^\mu \rightarrow 0} \int_{-1}^1 dx \, x \, G_3(x, \xi) = -J_q + \frac{1}{2} \Delta q = -L_q. \quad (14)$$

Here  $L_q$  is the quark orbital momentum contribution to the proton spin. In derivation we used Ji's sum rule [7]. We see that the distribution  $G_3$  is deeply related to the spin structure of the nucleon.

The second relation (13) is the non-forward generalization of the Efremov-Leader-Teryaev sum rule [16], it reduces to the ELT sum rule in the forward limit, because in the forward limit the SPD's  $\tilde{H}$  and  $\tilde{G}_2$  are reduced to the spin distributions  $g_1$  and  $g_2$  correspondingly.

As the final remark we note that the new functions  $G_i$  and  $\tilde{G}_i$  can be related to the leading twist functions  $H, \tilde{H}$  and  $E, \tilde{E}$  if one neglects the contributions of operators of type  $\bar{\psi}G\psi$ . The method of derivation is analogous to derivation of the Wandzura-Wilczek type of relations between twist-2 and twist-3 rho-meson distributions amplitudes [15]. Recently such relations were derived in ref. [13].

## Discussion

We have demonstrated that the handbag contribution to the DVCS amplitude up to the order  $1/Q$  depends not only on the leading twist skewed parton distributions  $(n \cdot F)$  and  $(n \cdot F^{(5)})$  but also on the additional functions  $F_{\rho_\perp}$  and  $F_{\rho_\perp}^{(5)}$ . These additional contributions are suppressed by only one power of the hard scale  $1/Q$  in the differential cross section. The

estimates of these contributions are important for the extraction of the leading twist skewed parton densities  $H, \tilde{H}$  and  $E, \tilde{E}$  from observables [17] and measuring the Ji's sum rule [7]. Also it is encouraging that the new functions can be also related to the spin structure of the nucleon, see eqs. (12,13). This can give additional possibility to extract quark orbital momentum from DVCS observables.

Note also that in our analysis we used the approximation  $\Delta_{\perp}^2 \ll Q^2$ , natural for the parton model. An account of the QCD evolution may lead effectively to a breakdown of this approximation and further complicate description of DVCS. Indeed, in the QCD ladder virtualities of the partons in the rungs of the ladder close to the nucleon are much smaller than  $Q^2$ . They are rather close to  $Q_0^2$ . Hence the accuracy of neglecting  $\Delta_{\perp}^2$  in these propagators is  $\sim \Delta_{\perp}^2/Q_0^2$ , not  $\sim \Delta_{\perp}^2/Q^2$ . The effects due to evolution of distributions  $F_{\mu\perp}$  and  $F_{\mu\perp}^{(5)}$  will be studied elsewhere.

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## References

- [1] J. Collins, L. Frankfurt, and M. Strikman, Phys. Rev. **D56** (1997) 2982.
- [2] X.-D. Ji, J. Osborne, Phys. Rev. **D58** (1998) 094018.
- [3] A. V. Radyushkin, Phys. Rev. **D56** (1997) 5524.
- [4] J.C. Collins, A. Freund, Phys. Rev. **D59** (1999) 074009.
- [5] J. Bartels and M. Loewe, Z.Phys. **C12** (1982) 263.
- [6] F.M. Dittes, D. Muller, D. Robaschik, B. Geyer, J. Horejsi Phys.Lett. **B209** (1988) 325; D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, J. Horejsi, Fortschr. Phys. **42** (1994) 101.
- [7] X. Ji, Phys. Rev. **D55** (1997) 7114.
- [8] I.V. Anikin, B. Pire, O.V. Teryaev, hep-ph/0003203.
- [9] J. Blümlein and D. Robaschik, hep-ph/0002071.
- [10] P.A.M. Guichon and M. Vanderhaeghen, Prog. Part. Nucl. Phys. **21** (1998) 125; M. Vanderhaeghen, P.A.M. Guichon, M. Guidal Phys. Rev. Lett. **80** (1998) 5064.
- [11] M.V. Polyakov and C. Weiss, Phys. Rev. **D60** (1999) 114017.

- [12] M. Diehl, *et al.* Phys. Lett. **B411** (1997) 193.
- [13] A. Belitsky, D. Müller, hep-ph/0007031.
- [14] V.Yu. Petrov, *et al.*, Phys. Rev. **D 57** (1998) 4325;  
M. Penttinen, M.V. Polyakov, K. Goeke, Phys. Rev. **D62** (2000) 014024.
- [15] P. Ball and V. M. Braun, Phys. Rev. **D54** (1996) 2186.
- [16] A.V. Efremov, O.V. Teryaev, E. Leader, Phys. Rev. **D55** (1997) 4307.
- [17] A.V. Belitsky, D. Müller, L. Niedermeir, and A. Schäfer, hep-ph/0004059.